

Hydrodynamics of Resonances

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Abstract

We derive system of equations describing fluidity of the medium consisting of non-relativistic particles with finite mass-widths. For that we use expressions for the kinetic Noether 4-current and the Noether energy-momentum tensor being conserved provided one uses self-consistent approximations to the gradient expanded Kadanoff-Baym equations. Kinetic coefficients entering equations of non-ideal hydrodynamics of resonances are obtained in terms of the real and imaginary parts of the self-energies within a relaxation time approximation.

1 Introduction

The appropriate frame for the description of non-equilibrium processes is the real-time formalism of quantum field theory, see [1–8] and refs. therein. This formalism finds now applications in many fields. The reason is the necessity of the dynamical description of broad resonances, as well as stable particles, which acquire a considerable width because of collision broadening. E.g., off-mass shell particles and resonances are extensively produced in heavy ion collisions.

Description of particles with broad widths requires development of self-consistent schemes with the conservation laws, being at least approximately satisfied [3,9–14]. Refs. [12] have shown that for the generalized kinetic equation in the so called Kadanoff-Baym (KB) form derived within the first space-time gradient approximation the conservation laws are exactly satisfied, provided one uses the Φ -derivable approximations, whereas in the so called Bottermans-Malfliet (BM) form they are approximately fulfilled within consistent first order gradient expansion. Such approaches permitted a numerical transport treatment of the off-shell dynamics of the particles in the matter [15–17,14].

Very close to the equilibrium at times much larger than the typical time for changes of kinetic quantities the kinetic description can be replaced by a more economical hydrodynamical description. The fluid-dynamical approach is fairly efficient for description of heavy-ion collisions in a broad collision energy range from SIS to RHIC energies (see e.g.

[18–20]). Recently an interest in the transport coefficient issue has been sharply increased in heavy-ion collision physics. Large values of the elliptic flow v_2 were observed at very high collision energies, at RHIC [21]. This can be understood as the created QGP behaves as a fluid with a small (but non-zero) value of the shear viscosity-to-entropy density ratio. The statement was confirmed with the help of non-ideal 2-dimensional hydrodynamical simulations, see [20]. In order to describe data in a broad energy range from SIS to SPS the 2-dimensional calculations are insufficient. The existing 3-dimensional hydrodynamical schemes use up to now ideal hydrodynamics. Although viscosity effects prove to be large, they are simulated indirectly with the help of an artificially introduced friction between different components of the liquid, see [19]. Refs. [22–25] argue that viscosity and heat conductivity effects are very important in the dynamical description of the first-order phase transitions. Search of possible manifestations of the critical endpoint in the nuclear matter phase diagram is one of the intriguing perspectives of the projects at FAIR, NICA and at low energy RHIC campaign. All mentioned stimulates significant interest to the development of the generalized fluid-dynamical description of the resonance matter.

In the modeling of the strongly interacting matter, interactions are often treated within the quasiparticle approximation although the width effects at least for some particle species can be very large. Refs. [26–28] calculated the shear and bulk viscosities of the hadron and quark phases within the quasiparticle approach in the relaxation time approximation in case where the effective masses of the constituents depend on the temperature and on the baryon density.

Although it is well known that the generalized kinetic approach naturally leads to expressions for the transport coefficients in terms of correlators like those discussed by Kubo, see [29], subsequent calculation of these correlators presents a complicated problem. To the best of our knowledge there are no consistent derivations of the kinetic coefficients from the generalized kinetic scheme which would be presented in terms of real and imaginary parts of the particle Green functions and self-energies, beyond the scope of the quasiparticle approximation. Estimates of the width effects on viscosities based on some reasonable conjectures have been done only recently [30,28].

In this paper using expressions for the kinetic Noether 4-current and energy-momentum tensor, which we introduce in sect. 2 following [10], we derive the generalized fluid-dynamical equations for the description of resonances (see sect. 3). In integral form these equations are presented in Appendix A. To be specific starting from sect. 3 we focus on description of non-relativistic dynamics. Then in sect. 4 we find an approximate solution of the kinetic equation in the BM form and in sect. 5 we derive transport coefficients expressed in terms of the self-energy functions. Some details necessary for calculations of kinetic coefficients are deferred to Appendices B-F. Thus we construct a consistent hydrodynamical approach for the description of particles with mass-widths.

2 Preliminaries

2.1 Generalized kinetic quantities in physical notations

To avoid the imaginary factors inherent in the standard Green function formulation one can introduce quantities, which are real and, in the quasi-homogeneous limit, positive, having clear physical meaning. So, instead of Green functions $G^{ij}(x, p)$ and self-energies $\Sigma^{ij}(x, p)$ ($i, j \in \{-+\}$) in the Wigner representation, we use the kinetic notation of Ref. [10], namely

$$\begin{aligned} F(x, p) &= A(x, p)f(x, p) = (\mp)\mathrm{i}G^{-+}(x, p), \\ \tilde{F}(x, p) &= A(x, p)[1 \mp f(x, p)] = \mathrm{i}G^{+-}(x, p), \end{aligned} \quad (2.1)$$

for the generalized Wigner functions F and \tilde{F} and the corresponding 4-phase-space distribution functions $f(x, p)$ and Fermi/Bose factors $[1 \mp f(x, p)]$. The upper sign corresponds to fermions, while the lower sign, to bosons. The space-time variables are $x \equiv x^\mu = (t, \mathbf{r})$, $t = \frac{1}{2}(t_1 + t_2)$, $\mathbf{r} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$, and the Fourier transformed $\xi = x_1 - x_2$ variables are $p \equiv p^\mu$.

The spectral function (spectral density) is

$$A(x, p) \equiv -2\mathrm{Im}G^R(x, p) = \tilde{F} \pm F, \quad (2.2)$$

and G^R is the retarded propagator. The spectral function satisfies the sum-rule

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} A(x, p) = 1, \quad (2.3)$$

for non-relativistic particles or

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} p_0 A(x, p) = 1, \quad (2.4)$$

for relativistic bosons.

The reduced gain and loss rates of the collision integral are defined as

$$\Gamma_{\mathrm{in}}(x, p) = (\mp)\mathrm{i}\Sigma^{-+}(x, p), \quad \Gamma_{\mathrm{out}}(x, p) = \mathrm{i}\Sigma^{+-}(x, p), \quad (2.5)$$

with the damping width

$$\Gamma(x, p) \equiv -2\text{Im}\Sigma^R(x, p) = \Gamma_{\text{out}}(x, p) \pm \Gamma_{\text{in}}(x, p), \quad (2.6)$$

where Σ^R is the retarded self-energy.

2.2 Kadanoff–Baym and Botermans–Malfliet forms of generalized kinetic equation

For simplicity consider the case, when there are no external fields. In terms of the generalized particle distribution function $F(x, p)$ the KB equation requires the form (so called KB form),

$$\mathcal{D}F(x, p) - \{\Gamma_{\text{in}}, \text{Re}G^R\} = C(x, p). \quad (2.7)$$

Here the differential drift operator is

$$\mathcal{D} = \left(v_\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p^\mu} \right) \partial_x^\mu + \frac{\partial \text{Re}\Sigma^R}{\partial x^\mu} \frac{\partial}{\partial p_\mu} \quad \text{with} \quad v^\mu = \frac{\partial}{\partial p_\mu} G_0^{-1}(p), \quad (2.8)$$

where $G_0^{-1}(p)$ is the Fourier transform of the inverse free Green function

$$G_0^{-1}(p) = \begin{cases} p^2 - m^2 & \text{for relativistic bosons} \\ p_0 - m - \mathbf{p}^2/(2m) & \text{for non-rel. fermions or bosons.} \end{cases} \quad (2.9)$$

In non-relativistic case we count the energy and the chemical potential from the mass. For relativistic bosons $v^\mu = 2p^\mu$ and for non-relativistic particles of the mass m ,

$$v^\mu \simeq (1, \mathbf{p}/m). \quad (2.10)$$

Symbol $\{\dots, \dots\}$ denotes the standard Poisson bracket,

$$\{f(x, p), \varphi(x, p)\} = \frac{\partial f}{\partial p^\mu} \frac{\partial \varphi}{\partial x_\mu} - \frac{\partial f}{\partial x^\mu} \frac{\partial \varphi}{\partial p_\mu}, \quad (2.11)$$

in covariant notation. Acting on an arbitrary function Ψ the drift operator \mathcal{D} yields

$$\mathcal{D}\Psi = \{M, \Psi\}, \quad (2.12)$$

with the “mass” function

$$M(x, p) = G_0^{-1}(p) - \text{Re}\Sigma^R(x, p). \quad (2.13)$$

The drift \mathcal{D} -term describes the particle drag flow. The commutator term in (2.7) has no clear physical meaning partially relating to the back flow and fluctuation effects. The collision term

$$C(x, p) = \Gamma_{\text{in}}(x, p)\tilde{F}(x, p) - \Gamma_{\text{out}}(x, p)F(x, p) \quad (2.14)$$

has the local part, $C_{\text{loc}}(x, p)$, and also the memory correction $C_{\text{mem}}(x, p)$. The latter appears if one includes into consideration the self-energy diagrams with more than two vertices, cf. [10]. Note that in the local approximation the collision term is charge (e.g., the baryonic number) and energy-momentum conserving by itself ¹

$$\text{Tr} \int \frac{d^4p}{(2\pi)^4} \begin{pmatrix} e \\ p^\mu \end{pmatrix} C^{\text{loc}} = 0. \quad (2.15)$$

The kinetic equation (2.7) is supplemented by the equation for the retarded Green function. Following [6] A is the algebraic function:

$$A(x, p) = \frac{\Gamma(x, p)}{M^2(x, p) + \Gamma^2(x, p)/4} + O(\partial_x^2), \quad (2.16)$$

up to second order gradient terms.

As can be seen from Eqs. (2.1), (2.6) and (2.14), the gain rate Γ_{in} differs from $F\Gamma/A$ only by corrections of the first order in the gradients

$$\Gamma_{\text{in}} = \Gamma F/A + C/A = \Gamma F/A + O(\partial_x), \quad (2.17)$$

since $C \sim O(\partial_x)$. This fact permits to neglect the correction $O(\partial_x)$ to Γ_{in} in the commutator term in the kinetic equation (2.7), as it leads to the second-order in the gradient terms. Thus upon substitution $\Gamma_{\text{in}} = \Gamma F/A$ in the commutator term [6] one arrives at the BM form of the kinetic equation,

$$\mathcal{D}F(x, p) - \left\{ \Gamma \frac{F}{A}, \text{Re}G^R \right\} = C(x, p), \quad (2.18)$$

which is equivalent to the KB form within the first-order gradient approximation, see [10] for details, (all terms $\propto O(\partial_x^2)$ are now omitted).

In terms of the four-phase-space occupation functions $f(x, p)$ the kinetic equation in the BM form can be rewritten as [10]

¹ Here and below the Tr means a sum over all possible internal degrees of freedom, like spin, and over possible particle species.

$$\frac{A^2\Gamma}{2} \left(\mathcal{D}f(x, p) - \frac{M}{\Gamma} \{ \Gamma, f \} \right) = C(x, p). \quad (2.19)$$

Opening the Poisson brackets we arrive at equation

$$\frac{A^2\Gamma}{2} \left[\left(v_\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \frac{\partial}{\partial x_\mu} + \left(\frac{\partial \text{Re}\Sigma^R}{\partial x^\mu} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial x^\mu} \right) \frac{\partial}{\partial p_\mu} \right] f = C. \quad (2.20)$$

With the help of (2.1), (2.6) the collision term can be expressed as

$$C = \Gamma_{\text{in}} A - A f \Gamma. \quad (2.21)$$

In the global thermal equilibrium

$$f_{\text{eq.}} = \frac{1}{\exp[(p_0 - \mu)/T] \pm 1}. \quad (2.22)$$

This equation holds also in the local thermal equilibrium in absence of the collective flow (at the velocity of the flow $\mathbf{U} = 0$),

$$f_{\text{l.eq.}}(\mathbf{U} = 0) = \frac{1}{\exp[(p_0 - \mu(t, \mathbf{r}))/T(t, \mathbf{r})] \pm 1}, \quad (2.23)$$

provided $T = T(t, \mathbf{r})$ and $\mu = \mu(t, \mathbf{r})$ are very smooth functions of (t, \mathbf{r}) (which vary on the space-time scales much larger than the kinetic scale). With distribution (2.23) the local part of the collision term vanishes; $C^{\text{loc}} = 0$.

2.3 Conservation of charge and energy-momentum

Transport equation (2.7) weighted either with the charge e , or with the 4-momentum p^ν , integrated over momentum and summed over internal degrees of freedom and particle species (Tr) gives rise to the charge or energy-momentum conservation laws with the density of the Noether 4-current and the Noether energy-momentum tensor [10]

$$j^\mu(x) = e \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v^\mu F(x, p), \quad (2.24)$$

$$\Theta^{\mu\nu}(x) = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v^\mu p^\nu F(x, p) + g^{\mu\nu} (\mathcal{E}^{\text{int}}(x) - \mathcal{E}^{\text{pot}}(x)) \equiv \Theta_{\text{kin}}^{\mu\nu} + \Theta_{\text{pot}}^{\mu\nu}. \quad (2.25)$$

Here

$$\mathcal{E}^{\text{int}}(x) = \langle -\widehat{\mathcal{L}}^{\text{int}}(x) \rangle \quad (2.26)$$

is the interaction energy density and

$$\mathcal{E}^{\text{pot}} = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} [\text{Re} \Sigma^R F + \text{Re} G^R \Gamma_{\text{in}}] = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} G_0^{-1}(p) A f \quad (2.27)$$

is the potential energy density. The first term in the squared brackets in the first equality of (2.27) complies with quasiparticle expectations, namely mean potential times density, the second term displays the role of fluctuations in the potential energy density.

For specific interactions with the same number α of field operators attached to any vertex of $\widehat{\mathcal{L}}^{\text{int}}$, one simply deduces [9,10,31]

$$\mathcal{E}^{\text{int}}(x, p) = \frac{2}{\alpha} \mathcal{E}^{\text{pot}}(x, p). \quad (2.28)$$

For two-body non-relativistic interaction and for relativistic boson ϕ^4 theory one gets $\alpha = 4$. For a theory with two single-flavor fermions interacting via one-flavor boson (with coupling $\Psi_f^\dagger \Psi_f (\phi_b + \phi_b^\dagger)$) one obtains

$$\epsilon_{\text{int}} = \frac{2}{\alpha} (\epsilon_{\text{pot}}^f + \epsilon_{\text{pot}}^b) = \frac{2}{\alpha_f} \epsilon_{\text{pot}}^f = \frac{2}{\alpha_b} \epsilon_{\text{pot}}^b, \quad \alpha = 3, \quad \alpha_f = 2, \quad \alpha_b = 1. \quad (2.29)$$

For a theory where two fermions with different flavors interact via one-flavor boson, one finds

$$\epsilon_{\text{int}} = 2\epsilon_{\text{pot}}^{f_1} = 2\epsilon_{\text{pot}}^{f_2} = 2\epsilon_{\text{pot}}^b. \quad (2.30)$$

For relativistic particles the energy-momentum tensor (2.25) is symmetric, i.e. $\Theta^{\mu\nu} = \Theta^{\nu\mu}$. For non-relativistic particles expression for the energy-momentum tensor is constructed from the relativistic expression with the help of the expansion, where now $p_0 \simeq m + p_0^{\text{n.-rel}}$ and $|p_0^{\text{n.-rel}}| \ll m$. As follows from (2.25), the non-relativistic value Θ^{0i} only approximately coincides with Θ^{i0} , provided v^μ is given by Eq. (2.10).

Ref. [10] has demonstrated that the conservation laws hold in the form

$$\partial_\mu j^\mu(x) = 0, \quad \partial_\mu \Theta^{\mu\nu}(x) = 0, \quad (2.31)$$

provided all the self-energies are Φ -derivable that we further assume. The latter means

that so called consistency conditions are fulfilled:

$$i\text{Tr} \int \frac{d^4p}{(2\pi)^4} [\{\text{Re}\Sigma^R, F\} \{\text{Re}G^R, \Gamma_{\text{in}}\} + C] = 0, \quad (2.32)$$

for the conserved current and

$$\partial^\nu (\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}) = \text{Tr} \int \frac{p^\nu d^4p}{(2\pi)^4} [\{\text{Re}\Sigma^R, F\} - \{\text{Re}G^R, \Gamma_{\text{in}}\} + C] \quad (2.33)$$

for the energy-momentum tensor.

All the properties of the KB-form of the kinetic Eq. (2.7) within a Φ -derivable approximation also transcribe to the BM-form, Eq. (2.18), through the substitution $\Gamma_{\text{in}} = \Gamma F/A$ in the consistency relations. However in difference with the KB form of the kinetic equation (2.7), where the Noether current (2.24) and the Noether energy-momentum tensor (2.25) are exactly conserved, with the BM form, Eq. (2.18), the conservation laws of the Noether current and the Noether energy-momentum tensor are only approximately fulfilled (up to higher order gradients), see the proof in [12]. For systems in the thermal equilibrium, expressions for thermodynamic quantities, being thermodynamically consistent with expressions for the Noether current and energy-momentum, can be found in [31].

3 Derivation of hydrodynamic equations for resonances

Below we follow the standard text-book procedure for the derivation of the system of equations of the fluid dynamics [32,33], although with a specifics that there is no strict dispersion relation between the energy and the momentum for broad resonances. To be specific and to easier clarify the physical meaning of different terms entering the system of equations of the fluid dynamics we further restrict ourselves by consideration of non-relativistic fermions or bosons.

3.1 Transformation between the laboratory and local rest frames and averaging procedure in 4-momentum space

Let in the laboratory frame (labeled by ') the given fluid element moves with the velocity \mathbf{U} . In the first approximation we assume that, although the system as a whole is in non-equilibrium, it can be sub-divided on macroscopic but physically small volumes, where the state can be considered as the equilibrium one. Now let us consider $\mathbf{U} = \mathbf{U}(t, \mathbf{r})$, as a smooth function of (t, \mathbf{r}) , being interpreted as the velocity of the center of inertia of a physically small fluid volume, i.e. the local velocity of the macroscopic motion of the

fluid. Since $p^\mu x_\mu$ is invariant, the energy and momentum of the particle in the laboratory frame and in the local rest frame are connected by relations

$$p'_0 \simeq p_0 + m\mathbf{v}\mathbf{U} + \frac{m\mathbf{U}^2}{2}, \quad \mathbf{p}' \simeq \mathbf{p} + m\mathbf{U}. \quad (3.1)$$

The spectral density in the local rest frame is

$$A = A_{\text{l.eq.}}(x, p_0, \mathbf{p}^2), \quad (3.2)$$

and the local equilibrium distribution function $f_{\text{l.eq.}}(x, p_0)$ is given by Eq. (2.23), p_0 is independent variable not related to the momentum.

After the variable shift (3.1), the $\text{Tr} \int \frac{d^4 p'}{(2\pi)^4} A' f' p'_i$ acquires the form²

$$\text{Tr} \int \frac{d^4 p'}{(2\pi)^4} A' f' p'_i = \rho U_i + \text{Tr} \int \frac{d^4 p}{(2\pi)^4} A f p_i = \rho U_i, \quad (3.3)$$

where it is used that Af may depend on \mathbf{p} only in \mathbf{p}^2 combination and thus

$$\text{Tr} \int \frac{d^4 p}{(2\pi)^4} A f p_i = 0, \quad (3.4)$$

demonstrating that the relative velocity of the chaotic thermal motion in the local equilibrium in the rest frame is zero.

The value

$$\rho = m \text{Tr} \int \frac{d^4 p}{(2\pi)^4} A f = m \bar{f} \quad (3.5)$$

can be interpreted as the mass density. Here we defined

$$\bar{\psi} = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} A \psi \quad (3.6)$$

for an arbitrary function ψ . Using the sum-role (2.3) we can rewrite (3.6) following the standard procedure of the averaging:

$$\bar{\psi} = \text{Tr} \int \frac{d^3 p}{(2\pi)^3} \left[\int \frac{dp_0}{2\pi} A \psi / \int \frac{dp_0}{2\pi} A \right]. \quad (3.7)$$

² We use Latin indices for the space-vector components.

Note that in the quasiparticle approximation,

$$A_{\text{q.p.}}(p_0, \mathbf{p}^2) = Z_0(2\pi)\delta(p_0 - \epsilon_p), \quad Z_0 = \left[v_0 - \frac{\partial \text{Re}\Sigma^R}{\partial p_0} \right]^{-1}, \quad (3.8)$$

where ϵ_p is the root of the relation $M(\Gamma \rightarrow 0) = 0$, the sum-rule (2.3) is not satisfied, and (3.6) and (3.7) are not equivalent. With (3.8) Eq. (3.6) yields

$$\text{Tr} \int \frac{d^4 p}{(2\pi)^4} A_{\text{q.p.}} \psi = \text{Tr} \int Z_0(p_0 = \epsilon_p) \psi[\epsilon(\mathbf{p}), \mathbf{p}] \frac{d^3 p}{(2\pi)^3}. \quad (3.9)$$

We also point out that the quasiparticle effective 4-current and the kinetic term in the energy-momentum tensor

$$j_{\text{q.p.}}^\mu = e \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \left(v^\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p_\mu} \right) A_{\text{q.p.}} f, \quad (3.10)$$

$$(\Theta_{\text{q.p.}}^{\text{kin}})^{\mu\nu} = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \left(v^\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p_\mu} \right) p^\nu A_{\text{q.p.}} f \quad (3.11)$$

differ from the Noether ones by the presence of the extra quasiparticle normalization factors $v_\mu \rightarrow \left(v_\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p^\mu} \right)$ arising from the interaction terms. Using (3.8) we see that the normalization factor is cancelled out in the definition of the density ρ and its presence results in the appearance of the effective mass instead of the bare mass in the expression for \mathbf{j} . Further to describe resonances we use the definition (3.6), rather than (3.7), and the Noether quantities for the current and the energy-momentum tensor.

3.2 Continuity equation

Presenting (2.24) in the laboratory frame, then doing the variable shift (3.1) and using first Eq. (2.31) and Eq. (3.4) we arrive at the continuity equation

$$\partial_t \rho + \text{div}(\rho \mathbf{U}) = 0, \quad (3.12)$$

which has the standard form in these variables. The value $\mathbf{j} = \rho \mathbf{U}$ is the density of the 3-mass-flow. In the local rest frame, where $\mathbf{U} = 0$, Eq. (3.12) is rewritten as

$$\partial_t \rho + \rho \text{div} \mathbf{U} = 0. \quad (3.13)$$

The continuity equation in the integral form is presented in Appendix A (see Eq. (6.1) there).

3.3 Momentum flow. Navier-Stokes equation

Taking $\nu = i$ ($i = 1, 2$ or 3) component in Eq. (2.25) for the energy-momentum tensor, we present the second Eq. (2.31) as ³

$$\frac{\partial(\overline{mf'v'_i})}{\partial t} + \frac{\partial(\overline{mv'_i v'_k f'})}{\partial x_k} = (F_i^{\text{int}})', \quad (3.14)$$

where $\mathbf{F}^{\text{int}} = -\nabla(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})$ can be interpreted as an internal force. After the variable shift (3.1), this equation is rewritten as

$$\frac{\partial(\rho U_i)}{\partial t} + \frac{\partial(\overline{mf v_i v_k})}{\partial x_k} + \frac{\partial(\rho U_i U_k)}{\partial x_k} = F_i^{\text{int}}, \quad (3.15)$$

where we used equation

$$\text{Tr} \int \frac{d^4 p'}{(2\pi)^4} A' f' p'_i p'_k = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} A f p_i p_k + m^2 U_i U_k \text{Tr} \int \frac{d^4 p}{(2\pi)^4} A f, \quad (3.16)$$

being obtained with the help of Eq. (3.4).

Using (2.25) we introduce the pressure according to the local-equilibrium relation

$$P = \frac{1}{3}(\Theta^{11} + \Theta^{22} + \Theta^{33})_{\text{l.eq.}} = P_{\text{kin}} + (\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})_{\text{l.eq.}}, \quad (3.17)$$

with the kinetic contribution

$$P_{\text{kin}} = \frac{2}{3} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \epsilon_p^0 A_{\text{l.eq.}} f_{\text{l.eq.}} = \frac{2}{3} \overline{\epsilon_p^0 f_{\text{l.eq.}}}, \quad \epsilon_p^0 = \frac{\mathbf{p}^2}{2m}. \quad (3.18)$$

We also introduce a symmetric tensor Π_{ik} as

$$\Pi_{ik}(\mathbf{U}) = P_{\text{kin}} \delta_{ik} - m \overline{f v_i v_k}, \quad (3.19)$$

and the vector

$$L_k(\nabla T) \equiv \Theta^{k0} = \overline{v_k p_0 f}, \quad (3.20)$$

³ Since further we deal with non-relativistic dynamics, for spatial components we use ordinary 3-dimensional notations, e.g., x_k means (x^1, x^2, x^3) .

the physical meaning of the latter is clarified below. These are straightforward generalizations of the standard Boltzmann expressions, which will allow us to derive equations of the fluid dynamics.

In the local equilibrium state in the local rest frame $f_{\text{l.eq.}}$ is the function of p_0 , $T(t, \mathbf{r})$ and $\mu(t, \mathbf{r})$, where T is the temperature and μ is the chemical potential, see Eq. (2.23). Thereby $m \overline{f_{\text{l.eq.}} v_i v_k} = \frac{m}{3} \overline{f_{\text{l.eq.}} v^2} \delta_{ik} = P_{\text{kin}} \delta_{ik}$ resulting in $\Pi_{ik} = 0$ and also $L_k = 0$. Using (3.19) and (3.17), and replacing $\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}$ to $(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})_{\text{l.eq.}} + \delta(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})$ we rewrite Eq. (3.15) in the standard form

$$\frac{\partial(\rho U_i)}{\partial t} + \frac{\partial(\rho U_i U_k)}{\partial x_k} \simeq -\frac{\partial P}{\partial x_i} + \frac{\partial \Pi_{ik}}{\partial x_k} + \delta F_i. \quad (3.21)$$

Eq. (3.21) represents the second Newton law for unit fluid volume. Integral form of this law is presented in Appendix A (see Eq. (6.2)), $\delta F_i = -\frac{\partial \delta(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})}{\partial x_i}$ can be interpreted as a force existing only in non-equilibrium (δF_i is zero in the local equilibrium state since $\delta(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}) = 0$). On the other hand the term δF_i can be presented as

$$\delta F_i = \frac{\partial \delta \Pi_{ik}}{\partial x_k}, \quad \delta \Pi_{ik} = -\delta_{ik} \delta(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}). \quad (3.22)$$

Note that the term (3.22) associated with the interaction in non-equilibrium state is usually ignored in practical calculations based on the Boltzmann kinetics, as a sub-leading term in the weak coupling limit and for dilute systems, see Eq. (5.22) of [34] and Eq. (2.1) of [27]. However there are no arguments to omit this term for a strong coupling and for dense systems.

In non-equilibrium states in the first approximation Π_{ik} should be proportional to the projections of the gradient of the components of the velocity vector, since in order a viscous friction of the near-by layers to appear, the velocities of the layers should be different. Thus the trace-less ($\Pi_{ik}^{(1)}$) and diagonal ($\Pi_{ik}^{(2)}$) parts of $\Pi_{ik} = \Pi_{ik}^{(1)} + \Pi_{ik}^{(2)}$, see Appendix A for more detail, yield

$$\Pi_{ik}^{(1)} = \eta W_{ik} \equiv \eta \left(U_{ik} - \frac{2}{3} \frac{\partial U_l}{\partial x_l} \delta_{ik} \right), \quad \Pi_{ik}^{(2)} = \zeta \frac{\partial U_l}{\partial x_l} \delta_{ik}, \quad \delta \Pi_{ik} = \delta \zeta \frac{\partial U_l}{\partial x_l} \delta_{ik}, \quad (3.23)$$

where $U_{ik} = \frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}$, η is the coefficient of the shear (first) viscosity and $\zeta + \delta \zeta$, of the bulk (second) viscosity. With these definitions Eq. (3.19) can be rewritten as

$$\delta \Theta_{ik}^{\text{kin}} \equiv \Theta_{ik}^{\text{kin}} - P_{\text{kin}} \delta_{ik} = -\zeta \frac{\partial U_l}{\partial x_l} \delta_{ik} - \eta W_{ik} = \Theta_{ik} - P \delta_{ik} - \delta_{ik} \delta(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}), \quad (3.24)$$

where we also used Eqs. (2.25) and (3.17).

For specific interactions with the same number α of field operators attached to any vertex, expression (3.22) is well defined by Eqs. (2.27), (2.28). E.g., for a theory with two single flavor fermions interacting with one flavor boson one has $\alpha_f = 2$ and $\alpha_b = 1$ that yields then no contribution to the bulk viscosity of the fermion sub-system, $\delta\zeta_f = 0$, but produces a contribution for the boson sub-system. On the other hand, one can use $\alpha = 3$ that allows to redistribute the potential–interaction energy terms between both sub-systems. This example teaches us that *the quantity $\delta\zeta$ is not uniquely defined for the sub-system of a multi-component system although it is uniquely defined for the system as a whole*. Possibility of a re-grouping of the interaction–potential energy between sub-systems may allow one to easier calculate $\delta\zeta = \sum_a \delta\zeta_a$. In many practically interesting situations a broad resonance appears, as a consequence of the interaction between other particle species. Those (other) particle species in many cases acquire much smaller widths than the given broad resonance and thereby they can be treated within the quasiparticle approximation. Thus it is convenient to relate the interaction–potential energy term to the quasiparticle species retaining broad resonances as quasi-free, see [31], since calculation of kinetic coefficients is easier done for quasiparticles.

With the help of Eqs. (3.23), (3.24) we rewrite Eq. (3.21) precisely in the Navier-Stokes form

$$\frac{\partial(\rho U_i)}{\partial t} + \frac{\partial(\rho U_i U_k)}{\partial x_k} + \frac{\partial P}{\partial x_i} - \eta \frac{\partial^2 U_i}{\partial x_k^2} - \left(\zeta + \delta\zeta + \frac{1}{3}\eta \right) \frac{\partial^2 U_k}{\partial x_i \partial x_k} = 0. \quad (3.25)$$

Now we may use an identity for an arbitrary function ψ :

$$\rho \left(\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x_k} \frac{\partial x_k}{\partial t} \right) = \frac{\partial(\psi \rho)}{\partial t} + \frac{\partial(\psi \rho U_k)}{\partial x_k}. \quad (3.26)$$

To derive this identity we used the continuity Eq. (3.12) and that $\frac{\partial x_k}{\partial t} = U_k$. With the help of this identity Eq. (3.21) can be rewritten as

$$\rho \frac{\partial U_i}{\partial t} + \rho U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial P}{\partial x_i} + \frac{\partial(\Pi_{ik} + \delta\Pi_{ik})}{\partial x_k}, \quad (3.27)$$

and as

$$\rho \frac{\partial U_i}{\partial t} = -\frac{\partial P}{\partial x_i} + \frac{\partial(\Pi_{ik} + \delta\Pi_{ik})}{\partial x_k} \quad (3.28)$$

in the local rest frame ($\mathbf{U} = 0$). The value

$$\delta P_{\text{n.eq.}} = \frac{1}{3} \left(\Theta^{11} + \Theta^{22} + \Theta^{33} \right)_{\text{n.eq.}} - \frac{1}{3} \left(\Theta^{11} + \Theta^{22} + \Theta^{33} \right)_{\text{l.eq.}}$$

$$= -\frac{1}{3}(\Pi_{ii}^{(2)} + \delta\Pi_{ii}) = -(\zeta + \delta\zeta)\text{div}\mathbf{U} \quad (3.29)$$

has the meaning of the correction to the pressure at small deviations from the local equilibrium (see Eq. (6.3) in Appendix A). Since $\zeta + \delta\zeta$ should be positive, the sign of the correction to the pressure depends on the sign of $\text{div}\mathbf{U}$. So, on the stage of expansion of the fireball in the heavy ion collision the non-equilibrium pressure is in reality smaller than the equilibrium one being used in the ideal hydrodynamics. This means that *realistic equilibrium equation of state might be stiffer than that allows to fit experimental data within ideal hydrodynamical simulations.*

If one sets $\Pi_{ik} + \delta\Pi_{ik} = 0$, one arrives at the Euler equation for the compressible ideal liquid

$$\rho \frac{\partial U_i}{\partial t} = -\frac{\partial P}{\partial x_i}. \quad (3.30)$$

In thermal equilibrium $\Pi_{ik} + \delta\Pi_{ik} = 0$, and the fluid of resonances is not viscous one (in spite of the production and absorption of resonances are included). For non-equilibrium systems the Euler equation may hold only approximately (provided η and ζ are very small).

3.4 Energy flow

Taking $\nu = 0$ component of the energy-momentum tensor (2.25) in the second Eq. (2.31) we obtain

$$\frac{\partial(\overline{v_0 p_0 f'})}{\partial t} + \frac{\partial(\overline{v'_k p'_0 f'})}{\partial x_k} = \partial_t(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}})'. \quad (3.31)$$

Using (3.1) and (3.19) we rewrite Eq. (3.31) as

$$\begin{aligned} & \frac{\partial(\overline{v_0 p_0 f})}{\partial t} + \frac{\partial(\rho \mathbf{U}^2/2)}{\partial t} + \frac{\partial(U_k \overline{p_0 f})}{\partial x_k} + \frac{\partial(\rho \mathbf{U}^2 U_k/2)}{\partial x_k} \\ &= -\frac{\partial L_k}{\partial x_k} + \partial_t(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}) - \frac{\partial(U_k P_{\text{kin}})}{\partial x_k} + \frac{\partial(U_i \Pi_{ik})}{\partial x_k}. \end{aligned} \quad (3.32)$$

As follows from Eq. (2.25), $\partial_t(\mathcal{E}^{\text{pot}} - \mathcal{E}^{\text{int}}) = \frac{\partial(\overline{v_0 p_0 f})}{\partial t} - \partial_t \mathcal{E}$, $\mathcal{E} \equiv \Theta^{00}$. Then with the help of Eq. (3.17) we rewrite Eq. (3.32) as

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial(\rho \mathbf{U}^2/2)}{\partial t} + \frac{\partial(U_k \mathcal{E})}{\partial x_k} + \frac{\partial(\rho \mathbf{U}^2 U_k/2)}{\partial x_k}$$

$$= -\frac{\partial L_k}{\partial x_k} - \frac{\partial(U_k P)}{\partial x_k} + \frac{\partial(U_i(\Pi_{ik} + \delta\Pi_{ik}))}{\partial x_k}. \quad (3.33)$$

This equation describes change of the energy with passage of time. Integral form of this equation is presented in Appendix A (see Eq. (6.3)).

In order to do Eq. (3.32) self-closed we need an expression for \mathbf{L} . Let us exploit the fact that the heat conductivity exists only in non-equilibrium states. Indeed, for the existence of the heat flow one needs a temperature gradient. Then in the first approximation

$$L_k = -\kappa \frac{\partial T}{\partial x_k}, \quad (3.34)$$

where κ is the coefficient of the heat conductivity depending on the properties of the matter.

Multiplying Eq. (3.21) by U_i and using that

$$U_i \left(\frac{\partial(\rho U_i)}{\partial t} + \frac{\partial(\rho U_i U_k)}{\partial x_k} \right) = \frac{1}{2} \left(\frac{\partial(\rho \mathbf{U}^2)}{\partial t} + \frac{\partial(\rho \mathbf{U}^2 U_k)}{\partial x_k} \right) \quad (3.35)$$

we obtain

$$\frac{\partial(\rho \mathbf{U}^2/2)}{\partial t} + \frac{\partial(\rho \mathbf{U}^2 U_k/2)}{\partial x_k} = U_i \frac{\partial(\Pi_{ik} + \delta\Pi_{ik} - P\delta_{ik})}{\partial x_k}. \quad (3.36)$$

The l.h.s. is the l.h.s. of the standard continuity equation, now for the kinetic energy of the fluid. In the r.h.s. of this equation we may recognize the work of the surface forces.

Using Eq. (3.36) from (3.33) we find

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial(U_k \mathcal{E})}{\partial x_k} = -\frac{\partial L_k}{\partial x_k} + \frac{\partial U_i}{\partial x_k} (\Pi_{ik} + \delta\Pi_{ik} - P\delta_{ik}). \quad (3.37)$$

Thereby, we recovered the standard form of the equation of the fluid dynamics describing the energy transport. In the frame $\mathbf{U} = 0$ in the local equilibrium we get

$$\frac{\partial \mathcal{E}}{\partial t} = -(\mathcal{E} + P) \frac{\partial U_k}{\partial x_k}. \quad (3.38)$$

Integral form of Eq. (3.37) is presented in Appendix A (see Eq. (6.4)).

3.5 Equation for evolution of the entropy

In the local equilibrium we may use the standard equilibrium expression

$$d\tilde{S} = \frac{1}{T}d\tilde{\mathcal{E}} + \frac{1}{T}Pd\tilde{V} = \frac{1}{T}d\tilde{\mathcal{E}} - \frac{P}{T}\frac{d\rho}{\rho^2}, \quad (3.39)$$

where tilde means that the quantity is related to the unit mass of the liquid. The sources of the entropy, which violate this relation have smallness of the second space-time gradient order [10]. Thus, $d\tilde{S} = 0$ in the first space-time gradient order and thereby

$$\frac{\partial(\mathcal{E}V)}{\partial V} = -P + O(\partial_x^2). \quad (3.40)$$

Note that in Eqs. (3.27), (3.37) we kept second gradient order terms $\propto \left(\frac{\partial U_i}{\partial x_k}\right)^2$ together with the first gradient order ones. Here we drop second gradient order terms compared to the zero order ones.

From (3.39) we get two equations

$$\frac{\partial\tilde{S}}{\partial t} = \frac{1}{T}\frac{\partial\tilde{\mathcal{E}}}{\partial t} - \frac{P}{T\rho^2}\frac{\partial\rho}{\partial t}, \quad U_k\frac{\partial\tilde{S}}{\partial x_k} = \frac{U_k}{T}\frac{\partial\tilde{\mathcal{E}}}{\partial x_k} - \frac{PU_k}{T\rho^2}\frac{\partial\rho}{\partial x_k}. \quad (3.41)$$

Summing up these two relations and then multiplying the result by ρ and using Eq. (3.26) we find

$$\frac{\partial(\rho\tilde{S})}{\partial t} + \frac{\partial(\rho\tilde{S}U_k)}{\partial x_k} = \frac{1}{T}\left[\frac{\partial(\rho\tilde{\mathcal{E}})}{\partial t} + \frac{\partial(\rho\tilde{\mathcal{E}}U_k)}{\partial x_k}\right] - \frac{P}{T\rho}\left(\frac{\partial\rho}{\partial t} + U_k\frac{\partial\rho}{\partial x_k}\right). \quad (3.42)$$

Using (3.37) in the first term in the r.h.s. and (3.12) in the second term we obtain

$$\frac{\partial(\rho\tilde{S})}{\partial t} + \frac{\partial(\rho\tilde{S}U_k)}{\partial x_k} = \frac{1}{T}\left((\Pi_{ik} + \delta\Pi_{ik})\frac{\partial U_k}{\partial x_i} - \frac{\partial L_k}{\partial x_k}\right). \quad (3.43)$$

We see that, if the r.h.s. of this equation were zero, we would get the continuity equation for the entropy. Thus the r.h.s. of Eq. (3.43) is the density of the entropy sources. Its integral form is presented in Appendix A (see Eq. (6.6)). In the local rest frame, $\mathbf{U} = 0$, Eq. (3.43) can be rewritten with the help of the continuity Eq. (3.13) as

$$\rho\frac{\partial\tilde{S}}{\partial t} = \frac{1}{T}\left[\frac{\eta}{2}U_{ik}^2 + (\zeta + \delta\zeta - \frac{2}{3}\eta)\left(\frac{\partial U_i}{\partial x_k}\right)^2 + \frac{\partial}{\partial x_k}\left(\kappa\frac{\partial T}{\partial x_k}\right)\right] > 0. \quad (3.44)$$

Only for $\eta, \zeta + \delta\zeta, \kappa \rightarrow 0$, the entropy is conserved,

$$\frac{\partial \tilde{S}}{\partial t} = 0. \quad (3.45)$$

Explicit expression for the Markovian entropy flow can be found in [10]. Also, on example of the first three diagrams of the Φ -functional for fermions interacting via a two-body potential, there was found the memory correction to the entropy in the local equilibrium.

4 Approximate solution of the generalized kinetic equation

Obtained above equations of fluid dynamics of resonance matter enter three kinetic coefficients η , ζ and κ which should be found from the solution of the Kadanoff-Baym equations. Kinetic coefficients can be derived assuming that deviations of the state of the moving fluid from the local equilibrium are small. To reproduce Π_{ik} in accordance with Eq. (3.19) we need to find the solution f of the Kadanoff-Baym equations (2.7), or (2.18), which are equivalent in the first gradient order. In order to find slightly inhomogeneous solutions of these equations with non-zero but small r.h.s. we present

$$F = A_{\text{l.eq.}}[f_{\text{l.eq.}}](f_{\text{l.eq.}} + \delta f) + \delta A[\delta f]f_{\text{l.eq.}}, \quad f = f_{\text{l.eq.}} + \delta f, \quad (4.1)$$

where in the local equilibrium state in terms of the variables of the laboratory frame $f_{\text{l.eq.}}$ is given by

$$f_{\text{l.eq.}} \simeq \frac{1}{\exp[(p'_0 - \mathbf{p}'\mathbf{U} - \mu(t, \mathbf{r}))/T(t, \mathbf{r})] \pm 1}, \quad (4.2)$$

and we suppose that $\mathbf{U}(t, \mathbf{r})$ is very small. By the notation $A_{\text{l.eq.}}[f_{\text{l.eq.}}]$ we stress that $A_{\text{l.eq.}}$ depends functionally on $f_{\text{l.eq.}}$.

As we have mentioned, the collision term [10] is subdivided in two pieces, the local term C^{loc} and the memory term C^{mem} . Following (2.21) one can demonstrate that in the local equilibrium the local collision term

$$C^{\text{loc}}[f_{\text{l.eq.}}] = A_{\text{l.eq.}}[f_{\text{l.eq.}}](\Gamma_{\text{in}}[f_{\text{l.eq.}}] - f_{\text{l.eq.}}\Gamma_{\text{l.eq.}}[f_{\text{l.eq.}}]) = 0. \quad (4.3)$$

To show this we use that $C^{\text{loc}}[f_{\text{l.eq.}}(p'_0, \mathbf{p}')] = C^{\text{loc}}[f_{\text{l.eq.}}(p_0, \mathbf{p})] = 0$ being obviously correct for $\mathbf{U}, \mu, T = \text{const.}$ Since the local part of the collision term does not depend on space-time gradients, the same is true for \mathbf{U}, μ, T being functions of t, \mathbf{r} .

Using (4.1) we find variation of the collision integral (2.14):

$$\delta C^{\text{loc}} = -A_{\text{l.eq.}} \Gamma_{\text{l.eq.}} \delta f + A_{\text{l.eq.}} \delta \Gamma^{\text{in}}[f] - A_{\text{l.eq.}} f_{\text{l.eq.}} \delta \Gamma[f] \equiv -A_{\text{l.eq.}} \tilde{\Gamma}_{\text{l.eq.}} \delta f. \quad (4.4)$$

Here there are terms containing $\delta \Gamma^{\text{in}}[\delta f]$ and $\delta \Gamma[\delta f]$ which only implicitly depend on δf . It is natural to expect that these dependencies might be weaker than that of the explicitly presented term. E.g. it is so if one considers the dynamics of the light particle admixture in the medium of heavy particles. Then the light particle self-energy can be considered as a function of the heavy particle distributions, whereas a dependence on the distribution of the light particle admixture can be neglected. Disregarding implicit dependence of $\delta \Gamma^{\text{in}}[\delta f]$ and $\delta \Gamma[\delta f]$ on the phase-space integrals of δf we get

$$\delta C \simeq -A_{\text{l.eq.}} \Gamma_{\text{l.eq.}} \delta f. \quad (4.5)$$

Such an approximation, or better to say ansatz, is in spirit of the known relaxation time approximation used in Boltzmann kinetics to describe near-equilibrium dynamics. Here all integrated distributions are replaced by their local equilibrium values and only not integrated one is allowed to have a non-equilibrium variation. We will name this approximation, *local relaxation time ansatz*. In a particular case of the Φ -functional presented up to two vertices the proof of the validity of this ansatz is presented in Appendix B. Also there we discuss a difference of the local relaxation time ansatz and the global one.

Treating the kinetic equation in the BM form (2.20) in the laboratory frame perturbatively we express δf through the local equilibrium quantities

$$\left(\frac{A^2 \Gamma}{2} \left[\left(v_\mu - \frac{\partial \text{Re} \Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \frac{\partial f}{\partial x_\mu} + \left(\frac{\partial \text{Re} \Sigma^R}{\partial x^\mu} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial x^\mu} \right) \frac{\partial f}{\partial p_\mu} \right] \right)_{\text{l.eq.}} - C_{\text{l.eq.}}^{\text{mem}} = -A_{\text{l.eq.}} \Gamma_{\text{l.eq.}} \delta f. \quad (4.6)$$

Here index "l.eq." indicates that all quantities are taken at local equilibrium in the laboratory frame, i.e. expressed in $'$ -variables. Also we may use that \mathbf{U} is small and put it zero after taking derivatives. Therefore one should keep only time derivatives in the second term ($\propto \frac{\partial f}{\partial p_\mu}$) in the l.h.s. of (4.6).

Since the collision term C^{mem} is of the first gradient order, we may consider it as functional of $f_{\text{l.eq.}}$. The explicit form of the memory collision term depends on what specific processes are studied. A specific example is discussed in Appendix C. The first non-vanishing diagram contributing to the term C^{mem} has at least three vertices [10]. Thus it contains an extra smallness at least for weak coupling. Γ and $\text{Re} \Sigma^R$ in the l.h.s. depend on $f_{\text{l.eq.}}$ only implicitly, i.e. through the phase-space integrals of $f_{\text{l.eq.}}$. This dependence is reflected in their dependence on $\mathbf{U}(\mathbf{r}, t)$, $\mu(\mathbf{r}, t)$ and $T(\mathbf{r}, t)$. Thus we arrive at equation for δf :

$$\left[\frac{A^2 \Gamma}{2} \left(v_\mu - \frac{\partial \text{Re} \Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \right]_{\text{l.eq.}} \frac{\partial}{\partial x_\mu} f_{\text{l.eq.}}$$

$$\begin{aligned}
& + \left[\frac{A^2 \Gamma}{2} \left(\frac{\partial \text{Re} \Sigma^R(p'_0 - \mathbf{p}' \mathbf{U}, \mathbf{p}' - m \mathbf{U}, \mu, T)}{\partial x^0} + \frac{M}{\Gamma} \frac{\partial \Gamma(p'_0 - \mathbf{p}' \mathbf{U}, \mathbf{p}' - m \mathbf{U}, \mu, T)}{\partial x^0} \right) \right]_{\text{l.eq.}} \\
& \times \frac{\partial f_{\text{l.eq.}}}{\partial p_0} - C_{\text{l.eq.}}^{\text{mem}} \simeq -A_{\text{l.eq.}} \Gamma_{\text{l.eq.}} \delta f.
\end{aligned} \tag{4.7}$$

Below we use Eq. (4.7) derived with the help of the local relaxation time ansatz (4.5). Using brief notations we will suppress index ' which indicated the laboratory frame. Following (4.2):

$$\begin{aligned}
\frac{\partial f_{\text{l.eq.}}}{\partial t} &= \frac{f_{\text{l.eq.}}(1 \mp f_{\text{l.eq.}})}{T} \left[\frac{(p_0 - \mu)}{T} \frac{\partial T}{\partial t} + \mathbf{p} \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mu}{\partial t} \right], \\
\nabla f_{\text{l.eq.}} &= \frac{f_{\text{l.eq.}}(1 \mp f_{\text{l.eq.}})}{T} \left[\frac{(p_0 - \mu)}{T} \nabla T + p_i \nabla U_i + \nabla \mu \right],
\end{aligned} \tag{4.8}$$

$$\frac{\partial f_{\text{l.eq.}}}{\partial p_0} = -\frac{f_{\text{l.eq.}}(1 \mp f_{\text{l.eq.}})}{T}, \quad \left(\frac{\partial f_{\text{l.eq.}}}{\partial \mathbf{p}} \right)_{\mathbf{U}=0} = 0, \tag{4.9}$$

and $\Sigma = \Sigma(p_0 - \mathbf{p} \mathbf{U}, \mathbf{p} - m \mathbf{U}, \mu, T)$. After taking derivatives we everywhere put $\mathbf{U} = 0$.

Then we express δf through the l.h.s. of Eq. (4.7) as

$$\begin{aligned}
\delta f &= -\frac{A f(1 \mp f)}{2 T} \left[\left(1 - \frac{\partial \text{Re} \Sigma^R}{\partial p_0} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p_0} \right) \frac{p_0 - \mu}{T} - \frac{\partial \text{Re} \Sigma^R}{\partial T} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial T} \right] \frac{\partial T}{\partial t} \\
&- \frac{A f(1 \mp f)}{2 T} \left[1 - \frac{\partial \text{Re} \Sigma^R}{\partial p_0} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p_0} - \frac{\partial \text{Re} \Sigma^R}{\partial \mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \mu} \right] \frac{\partial \mu}{\partial t} \\
&- \frac{A f(1 \mp f)}{2 T} \mathbf{p} \frac{\partial \mathbf{U}}{\partial t} \\
&- \frac{A f(1 \mp f)}{2 T} \left(\mathbf{v} + \frac{\partial \text{Re} \Sigma^R}{\partial \mathbf{p}} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \mathbf{p}} \right) \left(\frac{p_0 - \mu}{T} \nabla T + \nabla \mu + p_i \nabla U_i \right).
\end{aligned} \tag{4.10}$$

All quantities in the r.h.s. are taken at the local equilibrium. Not to complicate consideration we omitted contribution of C^{mem} . Demonstration how one can include contribution of C^{mem} is given in Appendix C.

5 Kinetic coefficients

To calculate shear viscosity we present [33]

$$\delta f = T^{-1} f_{\text{l.eq.}} g_{lh} U_{lh}, \quad (5.1)$$

for $i \neq k$. From (3.19) one has

$$\Pi_{ik} = -\frac{1}{T} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v_i p_k A_{\text{l.eq.}} f_{\text{l.eq.}} g_{lh} U_{lh} \equiv \frac{1}{2} \eta_{iklh} U_{lh}. \quad (5.2)$$

The quantities η_{iklh} form tensor of the rank 4, symmetric in indices ik and lh , being zero at the contraction with respect to the pair lh . Since the fluid is isotropic, this tensor is then expressed as: $\eta_{iklh} = \eta[\delta_{il}\delta_{kh} + \delta_{ih}\delta_{kl} - \frac{2}{3}\delta_{ik}\delta_{lh}]$. Thus we obtain

$$\eta = -\frac{1}{10T} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v_l p_h g_{lh} f_{\text{l.eq.}} A_{\text{l.eq.}}. \quad (5.3)$$

In order to calculate the bulk viscosity we present

$$\delta f = T^{-1} f_{\text{l.eq.}} g \text{div} \mathbf{U}. \quad (5.4)$$

Then we obtain

$$\zeta = -\frac{1}{3T} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} p_i v_i g f_{\text{l.eq.}} A_{\text{l.eq.}}, \quad (5.5)$$

$$\delta\zeta = -\frac{(1 - 2/\alpha)}{T} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} (p_0 - m - \frac{p^2}{2m}) g f_{\text{l.eq.}} A_{\text{l.eq.}}. \quad (5.6)$$

To derive (5.6) we used Eqs. (3.22), (3.23) and (2.27), (2.28). Performing variations following local relaxation time ansatz we omitted variations $\delta A[\delta f]$ since A depends on δf only implicitly. In the weak coupling limit and for the low densities one has $\delta\zeta \ll \zeta$, cf. free resonance case in [31].

The energy flux is given as $L_i = \Theta^{i0} = \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v_i p_0 A_{\text{l.eq.}} f$, cf. (3.20). We search

$$\delta f = g_i \frac{\partial T}{\partial x_i} f_{\text{l.eq.}}. \quad (5.7)$$

Then using Eq. (3.34) we find

$$\kappa = -\frac{1}{3} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} v_i p_0 g_i f_{\text{l.eq.}} A_{\text{l.eq.}}. \quad (5.8)$$

Substituting Eqs. (5.1), (5.4), (5.7) into Eq. (4.10) and using expressions (6.25), (6.26) derived in Appendix D we find⁴

$$g_{ik} = -\frac{Af(1 \mp f)}{2} \frac{p_i p_k}{m} \left[1 + \frac{\partial \text{Re} \Sigma^R}{\partial \epsilon_p^0} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \epsilon_p^0} \right], \quad (5.9)$$

$$g = -\frac{A(1 \mp f)}{2} \tilde{Z}_0^{-1} m I_\zeta, \quad (5.10)$$

$$\mathbf{g} = -\mathbf{v} \frac{A(1 \mp f)}{2T^2} \left(1 + \frac{\partial \text{Re} \Sigma^R}{\partial \epsilon_p^0} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \epsilon_p^0} \right) (p_0 - \mu - TS), \quad (5.11)$$

where S is the entropy per baryon,

$$I_\zeta = \frac{\mathbf{p}^2}{3mm^*} - \left[1 - \tilde{Z}_0 \left(\frac{\partial \text{Re} \Sigma^R}{\partial \mu} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \mu} \right) \right] \left(\frac{\partial P}{\partial \rho} \right)_{\Theta_{00}} - \left\{ \frac{p_0}{m} - \tilde{Z}_0 \left[\frac{T}{m} \left(\frac{\partial \text{Re} \Sigma^R}{\partial T} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial T} \right) + \frac{\mu}{m} \left(\frac{\partial \text{Re} \Sigma^R}{\partial \mu} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \mu} \right) \right] \right\} \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n. \quad (5.12)$$

Here we introduced the renormalization factor \tilde{Z}_0 and the ratio of the group velocity to the phase velocity:

$$\tilde{Z}_0 = \left(1 - \frac{\partial \text{Re} \Sigma^R}{\partial p_0} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p_0} \right)^{-1}, \quad \frac{v_{\text{group}}}{v_{\text{phase}}} \equiv \frac{m}{m^*} = \tilde{Z}_0 \left[1 + \frac{\partial \text{Re} \Sigma^R}{\partial \epsilon_p^0} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial \epsilon_p^0} \right], \quad (5.13)$$

$\mathbf{v}_{\text{phase}} = \mathbf{p}/m$. The value $\mathbf{v}_{\text{group}} \rightarrow d\epsilon_p/d\mathbf{p}$ for $\Gamma \rightarrow 0$, as it follows from the dispersion relation (6.35). Then the value m^* for low momenta has the meaning of the Landau (non-relativistic) effective mass ($v_{\text{group}} \rightarrow \mathbf{p}/m^*$). Deriving expression for the heat conductivity we also used that following (6.26) the terms $\propto \mathbf{v} \frac{\partial \mathbf{U}}{\partial t}$ appearing in (4.10) yield no contribution.

In order not to complicate consideration we omitted a contribution of the memory collision term to the kinetic coefficients, which form depends on the specific diagrams under consideration. Memory term can be calculated following the line shown in Appendix C.

Finally we obtain the following expressions for the kinetic coefficients:

⁴ Further to shorten notations we suppress index indicating local equilibrium.

$$\eta = \frac{1}{15} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \frac{A^2 \Gamma}{2} \tau_{\text{rel}} \frac{\mathbf{p}^4 f(1 \mp f)}{T m m^*}, \quad (5.14)$$

$$\zeta = \frac{1}{3} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \frac{A^2 \Gamma}{2} \tau_{\text{rel}} \frac{\mathbf{p}^2 f(1 \mp f)}{T} I_\zeta, \quad (5.15)$$

$$\delta\zeta = (1 - \frac{2}{\alpha}) \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \frac{A^2 \Gamma}{2} \tau_{\text{rel}} m (p_0 - m - \frac{\mathbf{p}^2}{2m}) \frac{f(1 \mp f)}{T} I_\zeta, \quad (5.16)$$

$$\kappa = \frac{1}{3} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \frac{A^2 \Gamma}{2} \tau_{\text{rel}} \frac{\mathbf{p}^2 f(1 \mp f)}{T} \frac{p_0 (p_0 - \mu - TS)}{m m^* T}, \quad (5.17)$$

where

$$\tau_{\text{rel}} = \tilde{Z}_0^{-1} \Gamma^{-1} \quad (5.18)$$

has the meaning of a relaxation time of the off-mass shell particle. Often one determines heat conductivity through the energy flux relative to the baryonic enthalpy, see [35]. The shift to the Eckart frame results in quadratic expression

$$\kappa_E = \frac{1}{3} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \frac{A^2 \Gamma}{2} \tau_{\text{rel}} \frac{\mathbf{p}^2 f(1 \mp f)}{T} \frac{(p_0 - \mu - TS)^2}{m m^* T}. \quad (5.19)$$

Our expressions for the kinetic coefficients present generalizations of expressions derived previously in different limit cases. Ref. [30] introduced expression for η in case of broad resonances at assumption that $\text{Re}\Sigma^R$ and Γ are approximately constants and therefore their derivatives are zero. Thus our expression (5.14) for η is the natural generalization of the result [30]. Ref. [28] conjectured expression for η for resonances in relativistic case at the assumption that Σ does not depend on p_0 and \mathbf{p} but may depend on μ and T . Equation for η used in [28] is the natural generalization of that given in Ref. [30].

The quasiparticle limit is reproduced if one replaces $\Gamma \rightarrow 0$ in the Green functions. For $\Gamma \rightarrow 0$ the value

$$\frac{A^2 \Gamma}{2} \rightarrow (2\pi) \delta(M). \quad (5.20)$$

Note that doing this replacement one should treat $\tau_{\text{rel}}^{\text{q.p.}}$ in (5.18) as a finite value. Here $\Gamma_{\text{q.p.}}$ should be calculated with the help of the quasiparticle Green functions. The quasiparticle

kinetic coefficients are obtained, if one uses expression for the energy-momentum tensor (3.11) instead of (2.25). Since η and ζ are derived from expression for the $(\Theta_{\text{q.p.}}^{\text{kin}})^{ik}$ there appears extra factor $Z_{\text{visc.}}^{-1} = 1 + \frac{\partial \text{Re}\Sigma^R}{\partial \epsilon_p^0}$. The quasiparticle expression for κ follows from the $(\Theta_{\text{q.p.}}^{\text{kin}})^{0i}$. Therefore there arises extra factor $Z_{\text{heat}}^{-1} = Z_0^{-1} = 1 - \frac{\partial \text{Re}\Sigma^R}{\partial p_0}$. After multiplication of expressions for the viscosities by $Z_{\text{visc.}}^{-1}$ and the replacement $Z_{\text{visc.}}^{-1} A^2 \Gamma / 2 \rightarrow (m/m^*) \delta(p_0 - \epsilon_p)$ and after multiplication of expression for the heat conductivity by Z_{heat}^{-1} and subsequent replacement $Z_{\text{heat}}^{-1} A^2 \Gamma / 2 \rightarrow \delta(p_0 - \epsilon_p)$, where ϵ_p is the solution of the dispersion equation

$$p_0 - m - \epsilon_p^0 - \text{Re}\Sigma^R(p_0, \mathbf{p}) = 0, \quad (5.21)$$

cf. Appendix E, we arrive at appropriate expressions for the quasiparticle kinetic coefficients:

$$\eta^{\text{q.p.}} = \frac{1}{15} \text{Tr} \int \frac{d^3 p}{(2\pi)^3} \tau_{\text{rel}}^{\text{q.p.}} \frac{\mathbf{p}^4 f(1 \mp f)}{T m_{\text{q.p.}}^{*2}}, \quad (5.22)$$

$$\zeta^{\text{q.p.}} = \frac{1}{3} \text{Tr} \int \frac{d^3 p}{(2\pi)^3} \tau_{\text{rel}}^{\text{q.p.}} \frac{\mathbf{p}^2 f(1 \mp f)}{T} I_{\zeta}^{\text{q.p.}}, \quad (5.23)$$

$$\kappa_{\text{E}}^{\text{q.p.}} = \frac{1}{3} \text{Tr} \int \frac{d^3 p}{(2\pi)^3} \tau_{\text{rel}}^{\text{q.p.}} \frac{\mathbf{p}^2 f(1 \mp f)}{T} \frac{(\epsilon_p - \mu - TS)^2}{m m_{\text{q.p.}}^* T}, \quad (5.24)$$

where $m^* = Z_0^{-1} Z_{\text{visc.}}$, $\tau_{\text{rel}}^{\text{q.p.}} = Z_0^{-1} \Gamma_{\text{q.p.}}^{-1}$ and

$$I_{\zeta}^{\text{q.p.}} = \frac{\mathbf{p}^2}{3 m_{\text{q.p.}}^{*2}} - \frac{m}{m_{\text{q.p.}}^*} \left[1 - Z_0 \frac{\partial \text{Re}\Sigma^R}{\partial \mu} \right] \left(\frac{\partial P}{\partial \rho} \right)_{\Theta_{00}} - \left\{ \frac{\epsilon_p}{m_{\text{q.p.}}^*} - Z_0 \left[\frac{T}{m_{\text{q.p.}}^*} \frac{\partial \text{Re}\Sigma^R}{\partial T} + \frac{\mu}{m_{\text{q.p.}}^*} \frac{\partial \text{Re}\Sigma^R}{\partial \mu} \right] \right\} \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n. \quad (5.25)$$

Obviously expressions for $\delta\zeta^{\text{q.p.}}$ and $\kappa^{\text{q.p.}}$ have similar forms. However one should bear in mind differences between definitions of purely quasiparticle terms (3.10), (3.11) and exact expressions (2.24), (2.25). Here in order to avoid double counting it is important to fix, whether one deals with the truncated kinetic equation for quasiparticles or with the full generalized kinetic equation consistent with the conservation of the Noether current and energy-momentum tensors.

At additional assumption that Σ does not depend on p_0 and \mathbf{p} but may depend on μ and T our expressions for $\eta^{\text{q.p.}}$ and $\zeta^{\text{q.p.}}$ coincide with those expressions derived previously in

[26,28] (after one re-writes the latter expressions in the non-relativistic limit). Additionally setting $\text{Re}\Sigma \rightarrow 0$ we reproduce the perturbation theory results, see [36].

Finally we should note that in the standard quasiparticle Fermi liquid theory one usually uses slightly different procedure to obtain transport coefficients, see [37]. Let us formulate the corresponding generalization of this procedure to the case of finite mass-widths. Let us use an ansatz that

$$F = A_{\text{l.eq.}}[f](f_{\text{l.eq.}} + \delta f), \quad (5.26)$$

where as above $\delta f = f - f_{\text{l.eq.}}$ but with $A[f]$ being functional of non-equilibrium f rather than of $f_{\text{l.eq.}}$. One can prove, see Appendix F, that replacing $F = A_{\text{l.eq.}}[f]f_{\text{l.eq.}}$ in the local collision term yields zero as it was for $F = A_{\text{l.eq.}}[f_{\text{l.eq.}}]f_{\text{l.eq.}}$. Now finding terms $\propto \delta f$ we should not vary A , $\text{Re}\Sigma^R$ and Γ , since they depend on exact f . Thus instead of (4.6) we get

$$\begin{aligned} & \frac{A^2\Gamma}{2} \left[\left(v_\mu - \frac{\partial \text{Re}\Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \frac{\partial f_{\text{l.eq.}}}{\partial x_\mu} + \left(\frac{\partial \text{Re}\Sigma^R}{\partial x^\mu} + \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial x^\mu} \right) \frac{\partial f_{\text{l.eq.}}}{\partial p_\mu} \right] \\ & - C^{\text{mem}}[A, f_{\text{l.eq.}}] = -A\Gamma\delta f. \end{aligned} \quad (5.27)$$

Although the r.h.s. of this kinetic equation looks more simple than that with the collision term (4.4), now all quantities except $f_{\text{l.eq.}}$ depend on unknown function f . In the quasiparticle approximation this unknown dependence on non-equilibrium distribution f is hidden in the values of the quasiparticle energies. However this is only an apparent simplification since the effective mass may depend on f . Simplifying one often ignores this dependence. Disregarding implicit dependence of $m^*[f]$ is in the spirit of the local relaxation time ansatz. Neglecting all implicit (functional) dependencies on f we actually do the same approximations as we have done above within local relaxation time ansatz.

6 Concluding remarks

In conclusion, starting with expressions for the Noether current and the energy-momentum tensor, as they follow from the gradient expanded Kadanoff-Baym equations, we derived equations describing the fluid dynamics of the non-relativistic system of resonances (particles with non-zero mass-widths). These equations, being expressed in appropriate variables, have the same form as standard equations of the fluid dynamics. The kinetic coefficients, the shear and bulk viscosities and the heat conductivity, are presented in terms of the self-energy functions and can be used beyond the scope of the ordinary quasiparticle approximation. We used a local relaxation time ansatz to get explicit expressions (in the Boltzmann kinetics a similar ansatz is called "the relaxation time approximation"). We found a contribution of the interaction-potential energy to the bulk viscosity

and discussed specifics of its interpretation for multi-component systems. We also demonstrated how one can include memory contributions. Finally, we discussed specifics of the quasiparticle limit.

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Appendix A. Equations of fluid dynamics in the integral form

Integrating the continuity Eq. (3.12) over a fixed volume V and using the Gauss theorem one obtains

$$\frac{d \int \rho d\mathbf{r}}{dt} = -m \int d\sigma_k j_k, \quad (6.1)$$

clearly showing that the change of the mass $\int \rho d\mathbf{r}$ in the given volume is determined by the mass-flow through the surface. In relativistic problems particles and anti-particles may annihilate in collisions producing radiation which should be taken into account. In non-relativistic systems resonances can be absorbed and produced in particle collisions. However their mass is assumed to be redistributed in accordance with the continuity equation, i.e. a small possible contribution of the radiation is ignored.

Integrating both parts of Eq. (3.21) over a fixed volume V with the help of the Gauss theorem we obtain

$$\frac{d(\int \rho U_i d\mathbf{r})}{dt} + \int d\sigma_k \rho U_i U_k = \int d\sigma_k (\Pi_{ik} + \delta \Pi_{ik} - P \delta_{ik}). \quad (6.2)$$

The l.h.s. is the change of the momentum in the volume V due to the change of the momentum at fixed point (first term) and due to the fluid motion through the surface (second term). The r.h.s. is the surface force $(\Pi_{ik} + \delta \Pi_{ik} - P \delta_{ik}) d\sigma_k$. In the local equilibrium $\Pi_{ik} + \delta \Pi_{ik} = 0$ and the surface force is perpendicular to the element $d\sigma$, being equal to $-P d\sigma$. In non-equilibrium states there appears a force $d\tilde{F}_i = (\Pi_{ik} + \delta \Pi_{ik}) d\sigma_k$, acting on the element of the surface square $d\sigma_k$ and having tangential components ($i \neq k$). These components are associated with the viscous friction. In order to explicitly separate them one presents Π_{ik} as the sum of two pieces, $\Pi_{ik} = \Pi_{ik}^{(1)} + \Pi_{ik}^{(2)}$, where $\Pi_{ik}^{(1)}$ is the traceless tensor $\Pi_{ik}^{(1)} = K_{ik} - \frac{1}{3} K \delta_{ik}$, $\text{Tr} \Pi_{ik}^{(1)} = 0$, $K = \text{Tr} K_{ik}$, and $\Pi_{ik}^{(2)} = N \delta_{ik}$, $\delta \Pi_{ik} = \delta N \delta_{ik}$. The force due to the diagonal part, $dF^{(2)} = (\Pi_{ik}^{(2)} + \delta \Pi_{ik}) d\sigma_k = (N + \delta N) d\sigma_i$, is orthogonal to the surface element $d\boldsymbol{\sigma}$ and has no tangential components. The traceless part $\Pi_{ik}^{(1)}$ results in an additional contribution to the non-equilibrium pressure $dF_i^{(1)} n_i = \Pi_{ik}^{(1)} n_i d\sigma_k =$

$(K_{ik} - \frac{1}{3}K\delta_{ik})n_i d\sigma_k$ and also in tangential forces of viscous friction. The $\Pi_{ik} + \delta\Pi_{ik}$ can be called the viscous stress tensor. The term Π_{ik} is reproduced from the Boltzmann kinetic equation, whereas the term $\delta\Pi_{ik}$ does not arise there.

Integrating Eq. (3.33) over some fixed volume V and using the Gauss theorem we obtain

$$\begin{aligned} & \frac{d}{dt} \left[\int d\mathbf{r} \left(\rho \mathbf{U}^2 / 2 + \mathcal{E} \right) \right] + \int d\sigma_k U_k \left(\rho \mathbf{U}^2 / 2 + \mathcal{E} \right) \\ &= \int (\Pi_{ik} + \delta\Pi_{ik} - P\delta_{ik}) U_i d\sigma_k - \int L_k d\sigma_k. \end{aligned} \quad (6.3)$$

In the l.h.s. one may recognize the change of the full energy in the volume V per unit time, as the consequence of the energy change at fixed point and due to the particle motion through the surface. This energy change consists of the kinetic energy and the internal one. The first term in the r.h.s. is the work of the surface forces, including the work of the pressure forces (the equilibrium pressure, P , and the non-equilibrium one, $\Pi = -\frac{1}{3}(\Pi_{ii}^{(2)} + \delta\Pi_{ii})$) and the work of viscous friction forces. The last term in (6.3) is the flow of the vector \mathbf{L} through the surface. It results in the change of the energy in the volume V even in absence of the viscous friction. Thereby one can interpret this term as the heat flow through the surface per unit time due to the heat conductivity. Thus the vector \mathbf{L} can be interpreted as the vector of the heat flow. The value $\int d\sigma_k U_k (\frac{1}{2}\rho \mathbf{U}^2 + \mathcal{E} + P)$ has the meaning of the heat content per unit volume.

After integration of (3.37) over the volume V we arrive at

$$\frac{d(\int d\mathbf{r} \mathcal{E})}{dt} + \int d\sigma_i U_i \mathcal{E} = - \int L_i d\sigma_i - \int P \frac{\partial U_i}{\partial x_i} d\mathbf{r} + \int d\mathbf{r} (\Pi_{ik} + \delta\Pi_{ik}) \frac{\partial U_i}{\partial x_k}. \quad (6.4)$$

This equation demonstrates that the reasons of the change of the internal energy in the volume V are the convection energy flow through the surface (second term in the l.h.s.), the heat flow (first term in the r.h.s.), the work of the pressure forces (second term in the r.h.s.) and the work of the viscous forces (third term in the r.h.s.). The viscosity always leads to a decrease of the mechanical energy and to an increase of the internal energy. Thereby the value $(\Pi_{ik} + \delta\Pi_{ik}) \frac{\partial U_i}{\partial x_k}$ should be positive in the non-equilibrium and zero in the equilibrium. Using Eqs. (3.23), we obtain

$$(\Pi_{ik} + \delta\Pi_{ik}) \frac{\partial U_i}{\partial x_k} = \frac{1}{2} \eta U_{ik}^2 + \left(\zeta + \delta\zeta - \frac{2}{3} \eta \right) \left(\frac{\partial U_k}{\partial x_k} \right)^2 \geq 0. \quad (6.5)$$

Integrating both parts of Eq. (3.43) over the volume V with the help of (6.5) we find

$$\frac{d(\int \rho \tilde{S} d\mathbf{r})}{dt} = - \int \rho \tilde{S} U_k d\sigma_k - \int \frac{L_k}{T} d\sigma_k$$

$$+ \int d\mathbf{r} \frac{1}{T} \left[\frac{\eta}{2} U_{ik}^2 + (\zeta + \delta\zeta - \frac{2}{3}\eta) \left(\frac{\partial U_i}{\partial x_k} \right)^2 \right]. \quad (6.6)$$

Thus, the change of the entropy consists of three parts. First term in the r.h.s. is the convection entropy flow due to the entropy transfer together with the fluid. Second term is the consequence of the heat conductivity, being the heat flow, in accordance with equation $d\tilde{S} = dQ/T$. This flow can be as positive as negative in dependence of the direction of the vector \mathbf{L} (of the temperature gradient). Third term describes the appearance of the entropy due to the viscous friction, being non-negative in agreement with the H theorem. This contribution is proportional to the velocity derivatives squared in agreement with the above assumption (3.39) of the quasi-equilibrium.

In the closed system there is no convection flow and the heat flow (with heat isolated walls), and the entropy grows owing to the viscous friction. If the system is in the local equilibrium, the second and third terms in the r.h.s. of Eq. (6.6) being zero and the entropy can only flow together with the fluid as the whole. The entropy of the isolated system remains then constant.

Appendix B. Relaxation time ansatz. Local relaxation time ansatz and its check on a simple example.

Generalized kinetic equation within relaxation time ansatz. Let us first show how one may arrive at the generalized kinetic equation within the relaxation time approximation, similar to what is done in ordinary Boltzmann kinetics. Expanding all quantities now *near the global equilibrium state*, rather than near the local equilibrium one, one obtains

$$\begin{aligned} & \left[\frac{A^2 \Gamma}{2} \left(v_\mu - \frac{\partial \text{Re} \Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \right]_{\text{eq.}} \frac{\partial \delta f}{\partial x_\mu} \\ & + \left[\frac{A^2 \Gamma}{2} \right]_{\text{eq.}} \left(\frac{\partial \delta \text{Re} \Sigma^R[\delta f]}{\partial x^0} + \frac{M_{\text{eq.}}}{\Gamma_{\text{eq.}}} \frac{\partial \delta \Gamma[\delta f]}{\partial x^0} \right) \frac{\partial f_{\text{eq.}}}{\partial p_0} - \delta C^{\text{mem}} = -A_{\text{eq.}} \tilde{\Gamma}_{\text{eq.}} \delta f, \end{aligned} \quad (6.7)$$

where now $\delta f = f - f_{\text{eq.}}$. Here in the l.h.s. we retained the terms which depend on δf but vanish for the equilibrium distribution. Then applying relaxation time ansatz, i.e. dropping variations of all quantities, which depend on δf only implicitly,⁵ we simplify Eq. (6.7) as

$$\left[\frac{A^2 \Gamma}{2} \left(v_\mu - \frac{\partial \text{Re} \Sigma^R}{\partial p^\mu} - \frac{M}{\Gamma} \frac{\partial \Gamma}{\partial p^\mu} \right) \right]_{\text{eq.}} \frac{\partial \delta f}{\partial x_\mu} \simeq -A_{\text{eq.}} \Gamma_{\text{eq.}} \delta f, \quad (6.8)$$

⁵ Within the relaxation time ansatz one omits δC^{mem} . Note that in the local equilibrium $C^{\text{mem}}[f_{\text{l.eq.}}] \neq 0$, see Eq. (6.15) in Appendix C, and one should keep this term performing calculations within the local relaxation time ansatz.

This equation can be treated as *the generalized kinetic equation in, as usually called, the relaxation time approximation.*

Local relaxation time ansatz. Now let us demonstrate the validity of *the local relaxation time ansatz*, when one considers *small deviations from the local equilibrium state.* This is a weaker assumption than the relaxation time ansatz just considered. Following the local relaxation time ansatz one keeps the implicit dependencies of space-time derivatives of the self-energies on $f_{\text{leq.}}(x, p)$ but one drops their implicit dependencies on δf .

To be specific consider a system of fermions interacting via a two-body potential $V = V_0\delta(x-y)$, and, for the sake of simplicity, disregard its spin structure, by relating spin and anti-symmetrization effects to a degeneracy factor d . For the first two diagrams within the Φ -derivable scheme (full Green functions and free vertices), the self-energy becomes

$$-i(\Sigma^{(1)} + \Sigma^{(2)}) = \text{Diagram 1} + \text{Diagram 2} \quad (6.9)$$

The self-energy here is presented up to two vertices. In this approximation there are no memory effects and the collision term $C^{\text{mem}} = 0$.

The collision term related to the second diagram (6.9) is given by

$$C^{(2)} = d^2 \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \left| \text{Diagram 2} \right|^2 \times \delta^4(p + p_1 - p_2 - p_3) (F_2 F_3 \tilde{F} \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F F_1). \quad (6.10)$$

In accordance with the local relaxation time ansatz, to find $\delta C^{(2)}$ we vary only F (and \tilde{F}) and do not vary F_1, F_2, F_3 since they are integrated. Also, varying F we vary f but do not vary A since the latter quantity depends on f only through the integrals. Thus we find

$$\delta C^{(2)} = -d^2 V_0^2 A(x, p) \delta f(x, p) \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \delta^4(p + p_1 - p_2 - p_3) \times (F_2 F_3 \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F_1). \quad (6.11)$$

On the other hand, using Eqs. (2.1), (2.6) and opening the structure of the second diagram (6.9) contributing to the width we obtain

$$\begin{aligned} \Gamma &= \Gamma_{\text{out}} - \Gamma_{\text{in}} = i\Sigma^{+-} - i\Sigma^{-+} \\ &= d^2 V_0^2 \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \delta^4(p + p_1 - p_2 - p_3) (F_2 F_3 \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F_1). \end{aligned} \quad (6.12)$$

Comparing (6.11) and (4.4) with the help of Eq. (6.12) we find that $\tilde{\Gamma} = \Gamma$. Thus, on this particular example we proved the validity of the local relaxation time ansatz.

Appendix C. Memory collision term with local equilibrium distributions

Let us continue to study example of a system of fermions interacting via a two-body potential studied in Appendix B. Now consider contribution of the collision term up to three vertices, [10].

For the first three diagrams within the Φ -derivable scheme, the self-energy becomes

$$\begin{aligned}
 -i\Sigma &= -i\left(\Sigma^{(1)} + \Sigma^{(2)} + \Sigma^{(3)}\right) = \\
 &= \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]}
 \end{aligned} \tag{6.13}$$

The local part of the collision term can be presented in the form

$$\begin{aligned}
 C^{(2)} + C_{\text{loc}}^{(3)} &= d^2 \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \left(\left| \text{[Diagram 4]} + \text{[Diagram 5]} \right|^2 - \left| \text{[Diagram 6]} \right|^2 \right) \\
 &\quad \times \delta^4(p + p_1 - p_2 - p_3) \left(F_2 F_3 \tilde{F} \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F F_1 \right),
 \end{aligned} \tag{6.14}$$

where all the vertices in the off-shell scattering amplitudes are of the same sign, say "−" for definiteness.

Also the collision term contains a *non-local* (memory) part due to the third diagram

$$\begin{aligned}
 C_{\text{mem}}^{(3)}(x, p) &= \left[\left(\Sigma_{+-}^{(3)} \right)_{\text{mem}}(x, p) G^{-+}(x, p) - G^{+-}(x, p) \left(\Sigma_{-+}^{(3)} \right)_{\text{mem}}(x, p) \right] \\
 &= \frac{i}{2} \int \frac{d^4 p'}{(2\pi)^4} \frac{1}{d} \left[\tilde{L}^{+-}(x; p' + p, p) - \tilde{L}^{-+}(x; p' + p, p) \right] \left\{ L^{+-}, L^{-+} \right\}_{p', x}.
 \end{aligned} \tag{6.15}$$

Here L^{jk} are the loops in the Wigner representation,

$$L^{jk}(x, p') = \int \frac{d^4 p''}{(2\pi)^4} \tilde{L}^{jk}(x; p' + p'', p'') \equiv iV_0 L_{\text{B}}^{jk}, \quad i, j = \{-, +\}, \tag{6.16}$$

$$\tilde{L}^{jk}(x; p' + p'') = diV_0 iG^{jk}(x, p' + p'') iG^{kj}(x, p''). \tag{6.17}$$

L_B^{jk} is the loop relativistic boson self-energy with unit vertices. To calculate contribution of the memory term to the kinetic coefficients we need $C^{\text{mem}}[f_{\text{1.eq.}}]$.

In the local equilibrium, as well as in the global equilibrium, the Green functions and self-energies obey simple relations

$$\mathcal{G}^{ik} = \begin{pmatrix} [1 \mp f_{\text{1.eq.}}] \mathcal{G}^R \pm f_{\text{1.eq.}} \mathcal{G}^A & \pm i f_{\text{1.eq.}} \mathcal{A} \\ -i [1 \mp f_{\text{1.eq.}}] \mathcal{A} & -[1 \mp f_{\text{1.eq.}}] \mathcal{G}^A \mp f_{\text{1.eq.}} \mathcal{G}^R \end{pmatrix}, \quad (6.18)$$

where $\mathcal{G}^{ik} = G_{\text{1.eq.}}^{ik}$ or $\Sigma_{\text{1.eq.}}^{ik}$ (in notation of [10]) and $\mathcal{A} = A$ or Γ , respectively, $\mathcal{G}^A = [\mathcal{G}^R]^*$.

Using these relations we express (6.15) for the local equilibrium distributions as

$$\begin{aligned} C_{\text{mem, 1.eq.}}^{(3)}(x, p) &= -\frac{A_{\text{1.eq.}}^{\text{F}}(x; p)}{2} \int \frac{d^4 p'}{(2\pi)^4} V_0^3 A_{\text{1.eq.}}^{\text{F}}(x; p' + p) \Gamma_{\text{1.eq.}}^{\text{B}}(x, p') \\ &\times [f_{\text{1.eq.}}^{\text{F}}(x, p) - f_{\text{1.eq.}}^{\text{F}}(x, p' + p)] \{f_{\text{1.eq.}}^{\text{B}}, \Gamma_{\text{1.eq.}}^{\text{B}}\}_{p', x} \\ &\equiv -A_{\text{1.eq.}}^{\text{F}}(x; p) \int \frac{d^4 p'}{(2\pi)^4} K_{\text{1.eq.}}(x; p', p) \{f_{\text{1.eq.}}^{\text{B}}, \Gamma_{\text{1.eq.}}^{\text{B}}\}_{p', x}. \end{aligned} \quad (6.19)$$

Since in the local equilibrium state all quantities are assumed to be known, further calculations of the memory contributions to the kinetic coefficients are straightforward. What we are able to say already without calculations is that in the weak coupling limit $C_{\text{mem, 1.eq.}}^{(3)}$ is small ($\propto V_0^3$) and can be neglected, thereby.

To further proceed we need to calculate $\{f_{\text{1.eq.}}^{\text{B}}, \Gamma_{\text{1.eq.}}^{\text{B}}\}_{p', x}$. Simplifying notations we will suppress index "1.eq.". Using (4.8), (4.9) and (6.25), (6.26) and that $\Gamma^{\text{B}} = \Gamma^{\text{B}}[T(x, t)]$ and $\mu^{\text{B}} = 0$ we find

$$\{f^{\text{B}}, \Gamma^{\text{B}}\}_{p', x} = \frac{f^{\text{B}}(1 + f^{\text{B}})}{T} \left(T \frac{\partial \Gamma^{\text{B}}}{\partial T} + p'_0 \frac{\partial \Gamma^{\text{B}}}{\partial p'_0} \right) \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n \text{div} \mathbf{U}. \quad (6.20)$$

We omitted terms $\propto \nabla(\mathbf{p}\mathbf{U})$ and $\propto \mathbf{p}\nabla T$ since they do not contribute to $C_{\text{mem}}^{(3)}$ disappearing after angular integrations. Thus the memory term contributes only to the bulk viscosity.

Following (4.7) one has $\delta f_{\text{mem}}^{\text{F}} = C_{\text{mem}}^{(3)}/(A^{\text{F}}\Gamma^{\text{F}})$ and from (5.5) we obtain

$$\delta C_{\text{mem}}^{(3)} = \frac{1}{3\Gamma^{\text{F}}(x; p)} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} \frac{\mathbf{p}^2}{m} K_{\text{1.eq.}}(x; p', p) \frac{f^{\text{B}}(x; p')[1 + f^{\text{B}}(x; p')]}{T}$$

$$\times \left(T \frac{\partial \Gamma^B(x; p')}{\partial T} + p_0' \frac{\partial \Gamma^B(x; p')}{\partial p_0'} \right) \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n. \quad (6.21)$$

Appendix D. Space-time dependence of thermodynamical quantities in the local equilibrium system

To find kinetic coefficients we use perturbative analysis considering their contributions as small. Therefore we exploit equations of motion (3.13), (3.30), (3.38) and (3.45). First we express pressure in different variables

$$P = P(n, \mathcal{E}), \quad P = P(\mu, T), \quad P = P(n, T). \quad (6.22)$$

Then using first of these equations and also Eqs. (3.13) and (3.38) we obtain

$$\frac{\partial P}{\partial t} = - \left\{ \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n \left[T \left(\frac{\partial P}{\partial T} \right)_\mu + \mu \left(\frac{\partial P}{\partial \mu} \right)_T \right] - \left(\frac{\partial P}{\partial n} \right)_\mathcal{E} \left(\frac{\partial P}{\partial \mu} \right)_T \right\} \text{div} \mathbf{U}. \quad (6.23)$$

On the other hand from the second Eq. (6.22) we find that

$$\frac{\partial P}{\partial t} = \left(\frac{\partial P}{\partial T} \right)_\mu \frac{\partial T}{\partial t} + \left(\frac{\partial P}{\partial \mu} \right)_T \frac{\partial \mu}{\partial t}. \quad (6.24)$$

With the help of the latter two expressions we obtain

$$\frac{\partial T}{\partial t} = - \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n T \text{div} \mathbf{U}, \quad \frac{\partial \mu}{\partial t} = - \left[\mu \left(\frac{\partial P}{\partial \mathcal{E}} \right)_n + \left(\frac{\partial P}{\partial n} \right)_\mathcal{E} \right] \text{div} \mathbf{U}. \quad (6.25)$$

With the help of the standard thermodynamic relation, Eq. (3.30) is rewritten as

$$\frac{\partial \mathbf{U}}{\partial t} = - \frac{\nabla P}{\rho} = - \frac{\nabla \mu}{m} - \frac{S \nabla T}{m} = 0, \quad (6.26)$$

since in the local equilibrium $\nabla P = 0$. Here S is the entropy per baryon.

Expressing $\tilde{S} = \tilde{S}(T, P)$ from (3.44) we find

$$\frac{\partial T}{\partial t} = - T c_P^{-1} \left(\frac{\partial \tilde{S}}{\partial P} \right)_T \frac{\partial P}{\partial t}, \quad c_P = T \left(\frac{\partial \tilde{S}}{\partial T} \right)_P. \quad (6.27)$$

From the third Eq. (6.22) and Eq. (3.13):

$$\frac{\partial P}{\partial t} = \left(\frac{\partial P}{\partial n} \right)_T n \text{div} \mathbf{U} + \left(\frac{\partial P}{\partial T} \right)_n \frac{\partial T}{\partial t}. \quad (6.28)$$

Using it and also (6.24) and (6.27) we obtain

$$\frac{\partial P}{\partial t} = -n \left(\frac{\partial P}{\partial n} \right)_T \left[1 + \frac{T}{c_p} \left(\frac{\partial \tilde{S}}{\partial P} \right)_T \left(\frac{\partial P}{\partial T} \right)_n \right]^{-1} \text{div} \mathbf{U}, \quad (6.29)$$

and thus

$$\left(\frac{\partial P}{\partial \mathcal{E}} \right)_n = -\frac{n}{c_P} \left(\frac{\partial \tilde{S}}{\partial P} \right)_T \left(\frac{\partial P}{\partial n} \right)_T \left[1 + \frac{T}{c_p} \left(\frac{\partial \tilde{S}}{\partial P} \right)_T \left(\frac{\partial P}{\partial T} \right)_n \right]^{-1}. \quad (6.30)$$

Appendix E. Dependence of the spectral function on its arguments in the laboratory frame

The spectral function (2.16) depends on p'_0 through specific combinations:

$$p'_0 - m - \epsilon_p^0 - \text{Re} \Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) + \tilde{\alpha} \mathbf{p}' \mathbf{U} + O(U^2), \quad \text{and} \quad (6.31)$$

$$\Gamma'_{\text{l.eq.}} = \Gamma_{\text{l.eq.}}(p'_0, \mathbf{p}'^2; \mu, T) - \tilde{\beta} \mathbf{p}' \mathbf{U} + O(U^2), \quad (6.32)$$

where

$$\tilde{\alpha} = \frac{\partial \text{Re} \Sigma_{\text{l.eq.}}^R}{\partial p'_0} + \frac{\partial \text{Re} \Sigma_{\text{l.eq.}}^R}{\partial \epsilon_p^0}, \quad \tilde{\beta} = \left(\frac{\partial \Gamma_{\text{l.eq.}}}{\partial p'_0} + \frac{\partial \Gamma_{\text{l.eq.}}}{\partial \epsilon_p^0} \right). \quad (6.33)$$

We used that $p_0 - m - \epsilon_p^0 = p'_0 - m - \epsilon_p^0 + O(U^2)$ since $p_0 \simeq p'_0 - \mathbf{p}' \mathbf{U}$, $\mathbf{p} \simeq \mathbf{p}' - m \mathbf{U}$ and therefore $\epsilon_p^0 \simeq \epsilon_p^0 - \mathbf{p}' \mathbf{U}$. Thus

$$A'_{\text{l.eq.}} \simeq A \left[p'_0 - m - \epsilon_p^0 - \text{Re} \Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) + \tilde{\alpha} \mathbf{p}' \mathbf{U}; \Gamma'_{\text{l.eq.}}(p'_0, \mathbf{p}'^2; \mu, T) \right]. \quad (6.34)$$

The argument $p'_0 - m - \epsilon_p^0 - \text{Re} \Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) + \tilde{\alpha} \mathbf{p}' \mathbf{U}$ can be further expanded in $p'_0 - \epsilon_p'$, where ϵ_p' is the root of the dispersion relation

$$p'_0 - m - \epsilon_p^0 - \text{Re} \Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) = 0, \quad (6.35)$$

which appears in the quasiparticle approximation. Note that in difference with the quasiparticle dispersion relation, here we do not assume that $\Gamma \rightarrow 0$. Then we obtain

$$p'_0 - m - \epsilon_{p'}^0 - \text{Re}\Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) \simeq \left(1 - \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}\right)_{p_0=\epsilon'_p} (p'_0 - \epsilon'_p). \quad (6.36)$$

Thus

$$\begin{aligned} & p'_0 - m - \epsilon_{p'}^0 - \text{Re}\Sigma_{\text{l.eq.}}^R(p'_0, \mathbf{p}'^2; \mu, T) + \bar{\alpha} \mathbf{p}' \mathbf{U} \\ & \simeq \left(1 - \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}\right)_{p'_0=\epsilon'_p} (p'_0 - \epsilon'_p + \bar{\alpha} \mathbf{p}' \mathbf{U}), \end{aligned} \quad (6.37)$$

where

$$\bar{\alpha} = \left[1 - \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}\right]_{p'_0=\epsilon'_p}^{-1} \left[\frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0} + \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial \epsilon_{p'}^0}\right]. \quad (6.38)$$

and in all quantities $p'_0 = \epsilon'_p$. In case when the dispersion equation has many roots ($\epsilon'_{p,a}$), the spectral function can be approximated by the sum of the corresponding terms. Note that the renormalization coefficients $\frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}$ and $\frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial \epsilon_{p'}^0}$ are absent only in the mean field approximation, when only tadpole diagrams are included. Finally

$$A'_{\text{l.eq.}} \simeq A \left[\left(1 - \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}\right)_{p'_0=\epsilon'_p} (p'_0 - \epsilon'_p + \bar{\alpha} \mathbf{p}' \mathbf{U}); \Gamma_{\text{l.eq.}} - \tilde{\beta} \mathbf{p}' \mathbf{U} \right]. \quad (6.39)$$

One can neglect the term $\propto \tilde{\beta}$ since it is $O(U\Gamma)$ provided both U and Γ are small. The quasiparticle limit expression (for $\Gamma \rightarrow 0$) becomes

$$A'_{\text{l.eq.}} \simeq \left(1 - \frac{\partial \text{Re}\Sigma_{\text{l.eq.}}^R}{\partial p'_0}\right)_{p'_0=\epsilon'_p}^{-1} (2\pi) \delta(p'_0 - \epsilon'_p + \bar{\alpha} \mathbf{p}' \mathbf{U}). \quad (6.40)$$

Appendix F. Local collision term for two presentations of F

Here we will show that the collision term $C^{\text{loc}} = 0$ not only for $F_{\text{l.eq.}}$ but also for F introduced by Eq. (5.26). To do this consider example of two-fermion interaction via two-body potential given in Appendix B. Consider three first diagrams (6.13) with full Green functions and free vertices. With (5.26) the local collision term (2.14) can be presented as

$$C^{(2)} + C_{\text{loc}}^{(3)} = d^2 \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \left(\left| \text{diagram 1} \right|^2 + \left| \text{diagram 2} \right|^2 - \left| \text{diagram 3} \right|^2 \right)$$

$$\begin{aligned}
& \times \delta^4(p + p_1 - p_2 - p_3) A(x, p) A(x, p_1) A(x, p_2) A(x, p_3) \\
& \times [f_{\text{l.eq.}}(x, p_2) f_{\text{l.eq.}}(x, p_3) (1 - f_{\text{l.eq.}}(x, p_1)) (1 - f_{\text{l.eq.}}(x, p)) \\
& - (1 - f_{\text{l.eq.}}(x, p_2)) (1 - f_{\text{l.eq.}}(x, p_3)) f_{\text{l.eq.}}(x, p_1) f_{\text{l.eq.}}(x, p)] .
\end{aligned} \tag{6.41}$$

Local equilibrium distributions (4.2) fulfill relation

$$\pm f_{\text{F/B}}^{\text{l.eq.}}(x, p + q) [1 \mp f_{\text{F/B}}^{\text{l.eq.}}(x, p)] = [f_{\text{F/B}}^{\text{l.eq.}}(x, p) \mp f_{\text{F/B}}^{\text{l.eq.}}(x, p + q)] f_{\text{B/F}}^{\text{l.eq.}}(x, q) \tag{6.42}$$

for fermions and bosons (F/B). With the help of this relation we may see that the term in the squared bracket in (6.41) is zero independent on the values of A . Thus $C^{\text{loc}} = 0$ for both distributions $F = A[f_{\text{l.eq.}}]f_{\text{l.eq.}}$ and $F = A[f]f_{\text{l.eq.}}$.

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